

Eddy currents computation in translational motion conductive plate of an induction heater with consideration of finite length extremity effects

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An electromagnetic model is proposed to compute translational motion eddy current in a conductive plate. The eddy currents are due to the movement of the plate in a dc magnetic field created by a PM inductor. Firstly, the magnetic field due to the PMs is computed in 3D where the iron yokes influence is considered thanks to the method of images. Then, the motional eddy currents are computed such that the edge effects are correctly taken into account through an iterative procedure which uses magnetic images. The computations are very fast and the obtained results are close to those issued from 3D FE method and from experiments.

Index Terms—eddy currents, induction heating device, method of images, linear motion.

I. INTRODUCTION

The studied induction heating device is shown in Fig.1. It is composed of two permanent magnets (PM)inductors with quasi Halbach magnetization arrangement. A conducting plate is placed between these two inductors and is subjected to a linear oscillatory motion. The geometric parameters of the studied device are given in the Table.I.

The geometry of the device is planar with finite dimensions. It has a strong magnetic end effects that need to be taken into account in the modeling[1],[2]. Such devices can be modeled by 3D Finite Element Method (FEM). However, this method have some drawbacks in terms of CPU time. In this work we develop a 3D modeling using the images theory to analyze the performances of the proposed induction heating device.

II. COMPUTATION METHOD

Using the images theory, the calculation of the heating power in the conducting plate is carried out in three steps:

- We compute the magnetic flux density created by the PMs.
- We calculate the eddy current density for an infinite conducting plate;
- For the conducting plate, we use the method of images to consider the finite boundaries of this plate.

A. Magnetic flux density

At the first stage, we calculate the magnetic field produced by each PM separately. Using the Amperian model, a PM block can be replaced by an equivalent current sheet placed on its lateral faces. In vacuum, the flux density is determined using the Biot-Savart law for which each current sheet is considered separately [3].

Superposition method is then used to determine the overall solution. To take into account the ferromagnetic yokes, the method of images is used to compute the magnetic field through an iterative procedure. The principle being the repetition of the computations in vacuum for different positions of the current sheets (the images of the current sheets) [3],[4],[5]. The number of images is theoretically infinite. However, only 10 iterations are needed to converge to a stable solution.

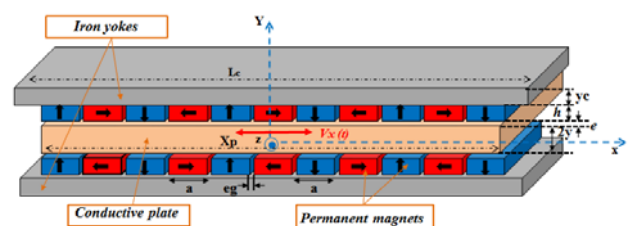


Fig.1. Geometry of the studied device.

Applying this method to the studied device, the 3D distribution of the flux density is obtained.

B. Eddy current expressions in the conductive plate

To obtain a homogeneous distribution of the temperature in the plate, its velocity should be low (typically 1m/s). Hence, the reaction field due to the induced currents in the plate can be neglected. The eddy currents in the conducting plate are then computed using

$$\overline{\text{curl}}(\vec{J}) = -\sigma \frac{\partial \vec{B}_m}{\partial t} \quad (1)$$

where \vec{B}_m is the flux density due to the magnets.

Obviously, (1) includes the conducting plate movement. Furthermore, the thickness of the conducting plate being relatively small compared to its depth, the eddy current density in the y-direction is negligible.

TABLE I
GEOMETRIC PARAMETERS OF THE INDUCTION HEATER

Symbol	Value	Designation
$2y_i$	15 mm	Conducting plate thickness
L_p	240 mm	Conducting plate length
Z_p	50 mm	Conducting plate width
e	3 mm	air gap
h	10 mm	Permanent magnet thickness
a	20 mm	Longitudinal PM width
b	20 mm	Transversal PM width
Z_a	50 mm	Permanent magnet length
y_c	20 mm	Iron yoke thickness

In these conditions, the eddy current density components in the x and z directions are given by:

$$\begin{cases} \frac{\partial J_z}{\partial y} = -V_x \sigma \frac{\partial B_x}{\partial x} \\ \frac{\partial J_x}{\partial y} = V_x \sigma \frac{\partial B_z}{\partial x} \end{cases} \quad (2)$$

In the case where the conducting plate has finite dimensions, the normal components of the eddy currents vanish on the lateral faces of the conducting plate. To satisfy these boundary conditions with the concept of images method [2],[6], an infinite multiplication of images can be introduced outside the plate (x and z directions). Thus, the actual eddy current density produced in a point of the plate is the superposition of the source eddy current density given by (2) (calculated in the case of an infinite plate) and the eddy current densities of the images. The total eddy current is written as

$$\vec{J}_{total} = \vec{J}_{source} + \sum \vec{J}_{images} \quad (3)$$

For the inductor magnets with normal (y) magnetization, the iterative image method leads then to the following expressions of the induced current density components J_z and J_x .

$$J_z = \frac{-V_x \sigma \mu_0 j_s}{4\pi} \sum_{x_n, z_n = -\infty}^{+\infty} (-1)^{x_n - z_n} \sum_{l=1}^2 \sum_{n=l:4}^{\infty} \sum_{i,j,k=0}^1 (-1)^{i+j+k} \times \left(\tan^{-1} \left(\frac{Y_1}{X} \right) - \tan^{-1} \left(\frac{ZY_1}{XR_1} \right) + \tan^{-1} \left(\frac{Y_2}{X} \right) - \tan^{-1} \left(\frac{ZY_2}{XR_2} \right) \right) \quad (4)$$

$$J_x = \frac{V_x \sigma \mu_0 j_s}{4\pi} \sum_{x_n, z_n = -\infty}^{+\infty} (-1)^{x_n - z_n} \sum_{l=1}^2 \sum_{n=l:4}^{\infty} \sum_{i,j,k=0}^1 (-1)^{i+j+k} \times \left(\ln(2Y_1 + 2R_1)(2Y_2 + 2R_2) \right) \quad (5)$$

Where:

$$\begin{aligned} X &= ((x_m + (x_n * X_p)) - (-1)^i a) + V_x t \\ Y_1 &= (y_m + j. h - H_n) \quad (\text{For images of PM located at } y > y_m) \\ Y_2 &= (-y_m + j. h + H_{n+2}) \quad (\text{For images of PM located at } y < y_m) \\ Z &= ((z_m + (z_n * Z_p)) - (-1)^k b) \end{aligned}$$

And

$$R_1 = \sqrt{X^2 + Y_1^2 + Z^2}; \quad R_2 = \sqrt{X^2 + Y_2^2 + Z^2}$$

With:

- x_m, y_m, z_m represent the coordinates of the point m where the flux density is computed,
- H_n is the height of the PM image.
- V_x is the linear velocity along x and t the time,
- j_s is the equivalent current sheet,
- σ is the electric conductivity of the plate,
- n represents the number of the images of PM,
- x_n and z_n represent respectively the number of images of the eddy currents along x and y .

A similar expression could also be written for the tangential (x) magnetization of PM. A stable solution is obtained with 4 images in the z direction and two images along x direction.

III. RESULTS

Fig.2 shows the waveforms of eddy current components at the middle of the plate.

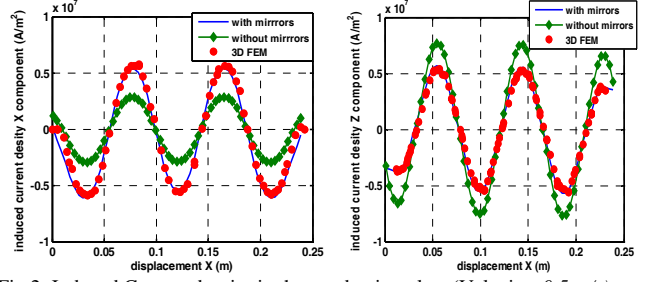


Fig.2: Induced Current density in the conducting plate (Velocity=0.5 m/s)

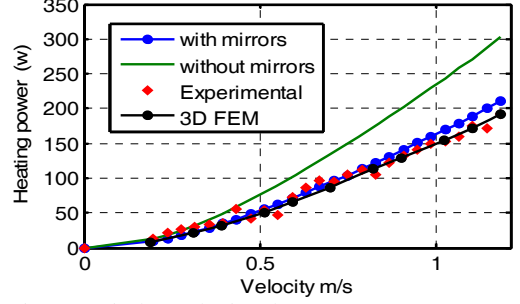


Fig.3: Heating power in the conducting plate.

A very good agreement is noticed between the proposed method and the FE results while the proposed method is much faster. The heating power dissipated by the Joule effect in the conducting plate is computed by the integration of the power density. The heating power calculated with the proposed is again in good accordance with FEM computation as well as with the measurements (Fig.3).

The computing time in 3D FEM using work station (48GO, 2 processors with 16core), is 196s, whereas the computing time in proposed method is about 50s on desktop PC(2GO, Intel core duo)

IV. CONCLUSION

The eddy currents in a conductive plate with consideration of finite length effects are computed in 3D by the method of images. The results are very close to those obtained with 3D FEM with the benefit of important reduction in the computation time. This is of great importance in any parametric or optimisation study where quick models are needed.

V. REFERENCES

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